



WASEDA University



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



THE UNIVERSITY OF TOKYO

# International Research Training Group 1529

## Mathematical Fluid Dynamics

# Autumn School and Workshop

Bad Boll, Germany

October 27 – 30, 2014

### Lecture Series

Peter Constantin, Princeton

PDE Problems of Hydrodynamic Origin

Yasunori Maekawa, Sendai

Analysis of Incompressible Flows in Unbounded Domains

László Székelyhidi, Leipzig

The  $h$ -Principle in Fluid Mechanics and Onsager's Conjecture

### Confirmed Speakers

K. Abe (Nagoya)

H. Abels (Regensburg)

D. Bothe (Darmstadt)

L. Brandolese (Lyon)

R. Danchin (Paris)

K. Disser (Berlin)

R. Farwig (Darmstadt)

E. Feireisl (Prague)

T. Hishida (Nagoya)

J. Kelliher (Los Angeles)

H. Koch (Bonn)

M. Kyed (Kassel)

M. Lopes Filho (Rio de Janeiro)

P. Maremonti (Naples)

P. Mucha (Warsaw)

Š. Nečasová (Prague)

J. Prüss (Halle)

M. Schoenbek (Santa Cruz)

F. Sueur (Paris)

R. Takada (Sendai)

W. Varnhorn (Kassel)

E. Zatorska (Warsaw)

### Organizers:

M. Hieber

H. Kozono

For further information please visit:  
[www.mathematik.tu-darmstadt.de/~igk/badboll2014](http://www.mathematik.tu-darmstadt.de/~igk/badboll2014)  
or contact: Verena Schmid, +49 6151 16 4694,  
[igk@mathematik.tu-darmstadt.de](mailto:igk@mathematik.tu-darmstadt.de)



Deutsche  
Forschungsgemeinschaft

DFG





---

## Contents

---

<b>1 Program</b>	<b>2</b>
<b>2 Short Talks</b>	<b>8</b>
<b>3 Abstracts</b>	<b>10</b>
<b>4 Participants</b>	<b>25</b>

# 1 Program

Autumn School "Mathematical Fluid Dynamics", Bad Boll, October 27 - 30, 2014				
Time	Monday	Tuesday	Wednesday	Thursday
8:55 - 9:00	Opening			
9:00h - 9:50h	Constantin	Székelyhidi	Maekawa	Constantin
10:00h - 10:50h	Maekawa	Constantin	Székelyhidi	Maekawa
11:00h - 11:30h	Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:30h - 11:55h	Brandolese	Abe	Zatorska	Takada
12:00h - 12:25h	Hishida	Kellihier	Lopes-Filho	Feireisl
12:30h - 14:00h	Lunch	Lunch	Lunch	Lunch
14:00h - 14:50h	Székelyhidi	Maekawa	Constantin	Székelyhidi
15:00h - 15:30h	6 short presentations	6 short presentations	9 short presentations	6 short presentations
15:00h - 15:45h				
15:30h - 16:15h	Coffee Break	Coffee Break	Coffee Break	Coffee Break
16:15h - 16:40h	Danchin	Koch	Tucsna	Abels
16:45h - 17:10h	Maremonti	Mucha	Necasová	Prüß
17:15h - 17:40h	Kyed	Disser	Lukácová	Closure
17:45h - 18:10h	Varnhorn	Ziegler		
18:30h	Reception		Dinner	
20:00h - 20:25h			Shibata	
20:30h - 20:55h			Bothe	

---

**Monday, 27. October 2014**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
<b>08:55-09:00</b>		OPENING
<b>09:00-09:50</b>	P. Constantin	<i>PDE Problems of Hydrodynamic Origin</i>
<b>10:00-10:50</b>	Y. Maekawa	<i>Analysis of Incompressible Flows in Unbounded Domains</i>
<b>11:00-11:30</b>		COFFE BREAK
<b>11:30-11:55</b>	L. Brandolese	<i>Blowup issues for water wave propagation in shallow water</i>
<b>12:00-12:25</b>	T. Hishida	<i>Stability of time-dependent Navier-Stokes flow and algebraic energy decay</i>
<b>12:30-14:00</b>		LUNCH
<b>14:00-14:50</b>	L. Székelyhidi	<i>The h-Principle in Fluid Mechanics and Onsager's Conjecture</i>
<b>15:00-15:30</b>		<i>Short Talks, see page 7</i>
<b>15:30-16:15</b>		COFFEE BREAK
<b>16:15-16:40</b>	R. Danchin	<i>Radiative flows and critical Besov spaces</i>
<b>16:45-17:10</b>	P. Maremonti	<i>A sort of continuous dependance on data for a suitable weak solution to the Navier-Stokes Cauchy problem</i>
<b>17:15-17:40</b>	M. Kyed	<i>Weak time-periodic solutions to the Navier-Stokes equations in the three-dimensional whole-space with a non-zero drift term: Asymptotic profile at spatial infinity</i>
<b>17:45-18:10</b>	W. Varnhorn	<i>On the non-homogeneous Navier-Stokes equations</i>
<b>18:30-21:00</b>		RECEPTION

---

---

**Tuesday, 28. October 2014**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
09:00-09:50	L. Székelyhidi	<i>The h-Principle in Fluid Mechanics and Onsager's Conjecture</i>
10:00-10:50	P. Constantin	<i>PDE Problems of Hydrodynamic Origin</i>
11:00-11:30		COFFEE BREAK
11:30-11:55	K. Abe	<i>The Navier-Stokes equations in a space of bounded functions</i>
12:00-12:25	J. Kelliher	<i>Serfati's approach to vortex patches</i>
12:30-14:00		LUNCH
14:00-14:50	Y. Maekawa	<i>Analysis of Incompressible Flows in Unbounded Domains</i>
15:00-15:30		<i>Short Talks, see page 7</i>
15:30-16:15		COFFEE BREAK
16:15-16:40	H. Koch	<i>Radiative flows and critical Besov spaces</i>
16:45-17:10	P. Mucha	<i>Mathematical cocktails: weak solutions</i>
17:15-17:40	K. Disser	<i>Asymptotic behaviour of a rigid body with a cavity filled by a viscous liquid</i>
17:45-18:10	M. Ziegler	<i>Computational Theory of Function Spaces</i>
18:30-20:00		DINNER
20:00-20:25	Y. Shibata	<i>On a global well-posedness of strong dynamics of Incompressible Nematic Liquid Crystals in <math>\mathbb{R}^N</math></i>
20:30-20:55	D. Bothe	<i>Modeling and analysis of multicomponent transport in fluid systems with ionic species</i>

---

---

**Wednesday, 29. October 2014**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
09:00-09:50	Y. Maekawa	<i>Analysis of Incompressible Flows in Unbounded Domains</i>
10:00-10:50	L. Székelyhidi	<i>The h-Principle in Fluid Mechanics and Onsager's Conjecture</i>
11:00-11:30		COFFEE BREAK
11:30-11:55	E. Zatorska	<i>Low Mach Number Limit for the Multicomponent Reactive Mixture Model</i>
12:00-12:25	M. Lopes Filho	<i>The limit of small viscosity and small elastic response for the second-grade fluid equations</i>
12:30-14:00		LUNCH
14:00-14:50	P. Constantin	<i>PDE Problems of Hydrodynamic Origin</i>
15:00-15:45		<i>Short Talks, see page 8</i>
15:45-16:15		COFFEE BREAK
16:15-16:40	M. Tucsnak	<i>Free and controlled particles in viscous incompressible flows</i>
16:45-17:10	Š. Nečasová	<i>Low Mach number limit and diffusion limit in a model of radiative flow</i>
17:15-17:40	M. Lukáčová	<i>Existence, uniqueness and approximation of some diffusive viscoelastic flows</i>
18:30-		DINNER

---

---

**Thursday, 30. October 2014**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
<b>09:00-09:50</b>	P. Constantin	<i>PDE Problems of Hydrodynamic Origin</i>
<b>10:00-10:50</b>	Y. Maekawa	<i>Analysis of Incompressible Flows in Unbounded Domains</i>
<b>11:00-11:30</b>		COFFEE BREAK
<b>11:30-11:55</b>	R. Takada	<i>Remarks on the Strichartz estimates for the rotating incompressible fluids</i>
<b>12:00-12:25</b>	H. Abels	<i>Well-Posedness and Stability for the Volume-Preserving Mean Curvature Flow with a Dynamic Contact Angle</i>
<b>12:30-14:00</b>		LUNCH
<b>14:00-14:50</b>	L. Székelyhidi	<i>The h-Principle in Fluid Mechanics and Onsager's Conjecture</i>
<b>15:00-15:30</b>		<i>Short Talks, see page 8</i>
<b>15:30-16:15</b>		COFFEE BREAK
<b>16:15-16:40</b>	E. Feireisl	<i>Regularity of weak solutions to the full Navier-Stokes-Fourier system</i>
<b>16:45-17:10</b>	J. Prüss	<i>Maxwell-Stefan Diffusion in Reactive Multicomponent Flows</i>
<b>17:15-17:40</b>		CLOSURE

---

---

## 2 Short Talks

---

---

### Monday, 27. October 2014

---

Time	Speaker	Title of Talk
15:00-15:05	T. Iwabuchi	<i>On the large time behavior of solutions for the critical Burgers equation</i>
15:05-15:10	C. Förster	<i>Wild initial data for the incompressible Euler equation</i>
15:10-15:15	M. Bolkart	<i>Strong Stability of the Stokes Semigroup in Spaces of Bounded Functions</i>
15:15-15:20	M. Kalousek	<i>Homogenization of a non-Newtonian flow through a porous medium</i>
15:20-15:25	C. Nobili	<i>Turbulent Convection at finite Prandtl number</i>
15:25-15:30	T. Kato	<i>Time decay estimates for dispersive equations</i>

---

---

### Tuesday, 28. October 2014

---

Time	Speaker	Title of Talk
15:00-15:05	A. Tarfulea	<i>Regularity and Finite-dimensional Attractor for the Critical Surface Quasigeostrophic Equation</i>
15:05-15:10	M. Saal	<i>Nonlinear Integro-Differential Equations</i>
15:10-15:15	C. Schmäche	<i>Non-smooth isometric embeddings of surfaces</i>
15:15-15:20	T. Seitz	<i>Flow identification from MRV measurements</i>
15:20-15:25	T. Suzuki	<i>Analyticity of semigroups generated by higher order elliptic operators in spaces of bounded functions on <math>C^1</math> domains</i>
15:25-15:30	E. Ushikoshi	<i>Hadamard variational formula for the eigenvalue of the Stokes equations</i>

---

---

**Wednesday, 29. October 2014**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
15:00-15:05	Y. Terasawa	<i>Sharp weighted maximal function estimate in some martingale setting</i>
15:05-15:10	J. Sauer	<i>Strong Stability of 2D Viscoelastic Poiseuille-type Flows, Part 1</i>
15:10-15:15	K. Schade	<i>Strong Stability of 2D Viscoelastic Poiseuille-type Flows, Part 2</i>
15:15-15:20	G. Takahashi	<i>Extension criterion on strong solutions to the Navier-Stokes equations</i>
15:20-15:25	G. Yanqiu	<i>Inertial manifolds for certain sub-grid scale alpha-models of turbulence</i>
15:25-15:30	M. Egert	<i>Elliptic boundary value problems on cylindrical domains</i>
15:30-15:35	B. She	<i>Numerical simulation of a diffusive Oldroyd-B model</i>
15:35-15:40	N. Mori	<i>Dissipative Timoshenko system versus Timoshenko-Cattaneo system</i>
15:40-15:45	S. Meyer	<i>On a two-phase weak Neumann problem</i>

---

**Thursday, 30. October 2014**

---

<b>Time</b>	<b>Speaker</b>	<b>Title of Talk</b>
15:00-15:05	H. Koba	<i>On fluid-flow on an evolving hypersurface</i>
15:05-15:10	P. Tolksdorf	<i>The Navier-Stokes equations on bounded Lipschitz domains</i>
15:10-15:15	T. Okabe	<i>Remark on the asymptotic expansion of the Navier-Stokes flow in the whole space</i>
15:15-15:20	A. Dalinger	<i>On the hydrodynamic behavior of a 1D system with next neighbour interactions</i>
15:20-15:25	H. Mizerová	<i>Regularity of weak solution to the diffusive Peterlin model</i>
15:25-15:30	D. Noboriguchi	<i>Stochastic Scalar Conservation Laws with Dirichlet boundary conditions</i>

---

## **The Navier-Stokes equations in a space of bounded functions**

Ken Abe

Nagoya University  
Furo-cho, Chikusa-ku, Nagoya, Japan  
abe.ken@math.nagoya-u.ac.jp

We consider blow-up rates of the Navier-Stokes flow for domains with boundaries subject to the non-slip boundary condition. It is known that the type I blow-up of the Navier-Stokes flow in the whole space or a half space is a minimum rate at which a singularity can develop. We develop an existence theorem on  $L^\infty$  for domains with curved boundaries by means of  $L^\infty$ -estimates for the Stokes semigroup and deduce the minimum blow-up rate.

## **Well-Posedness and Stability for the Volume-Preserving Mean Curvature Flow with a Dynamic Contact Angle**

Helmut Abels

University of Regensburg, Germany  
helmut.abels@mathematik.uni-regensburg.de

We will consider the dynamic stability of a spherical cap on a flat plane in three space dimensions with a dynamic boundary condition which is associated to an energy of the contact line between the spherical cap and the plane. Using a suitable parametrization we can apply results on maximal  $L^p$ -regularity for parabolic equations with dynamic boundary conditions and a generalized principle of linearized stability. To this end a careful analysis of the spectral properties of the linearized operator is needed.

---

# Modeling and analysis of multicomponent transport in fluid systems with ionic species

Dieter Bothe

Technische Universität Darmstadt,  
64289 Darmstadt, Germany  
bothe@csi.tu-darmstadt.de

Ionic species in fluid systems are subject to Coulomb forces due to the intrinsic electrical field generated by their charges. This leads to a drift contribution, the so-called electromigration, to the transport of the ions. We give a thermodynamically consistent derivation of electromigration based on the Maxwell-Stefan approach to multicomponent transport and discuss the wellposedness of several prototype models.

## Blowup issues for water wave propagation in shallow water

Lorenzo Brandolese

Université Lyon 1,  
43 bd. du 11 Novembre, 69622 Villeurbanne, France  
brandolese@math.univ-lyon1.fr

We study the formation of singularities in strong solutions of one-dimensional asymptotic models derived from the free-surface irrotational Euler equation in the shallow water and moderate amplitude regime. Such models include, e.g., the Camassa–Holm equation and the Degasperis–Procesi equation. We will provide a new blowup criterion that allow us to unify some of the earlier best known blowup results. The presented models are also relevant in the study of vibrations inside hyperelastic materials and in numerical regularizations of turbulent flows.

This is a partially joint work with Fernando Cortez.

### References

- [1] L. Brandolese, *Local-in-space criteria for blowup in shallow water and dispersive rod equations*, Comm. Math. Phys. **330**, N. 1 (2014), 401–414.
- [2] L. Brandolese, M. F. Cortez, *On permanent and breaking waves in hyperelastic rods and rings*, J. of Funct. Anal. **266** (2014), 6954–6987.

---

[3] L. Brandolese, *A Liouville theorem for the Degasperis-Procesi equation*, preprint, arXiv:1405.6675.

## **Radiative flows and critical Besov spaces**

Raphaël Danchin

Université Paris XII, France  
raphael.danchin@u-pec.fr

We consider a simplified model arising in radiation hydrodynamics. It is based on the barotropic Navier-Stokes system describing the macroscopic fluid motion, and the P1-approximation of the transport equation modeling the propagation of radiative intensity. Under a necessary and sufficient linear stability condition, we establish the global-in-time existence of strong solutions for small perturbations of a stable radiative equilibrium. Local existence is proved for any large data with critical regularity, and density bounded away from zero. This is a joint work with B. Ducomet. Dynamics which is based on the barotropic Navier-Stokes system describing the macroscopic fluid motion, and the so-called P1-approximation of the transport equation modeling the propagation of radiative intensity. We establish global-in-time existence of strong solutions for the associated Cauchy problem when initial data are close to a stable radiative equilibrium, and local existence for large data with no vacuum. We also discuss the low Mach number limit and various diffusive asymptotics.

## **Asymptotic behaviour of a rigid body with a cavity filled by a viscous liquid**

Karoline Disser

Weierstrass Institute,  
Mohrenstr. 39, 10117 Berlin,  
karoline.disser@wias-berlin.de

We discuss the system of equations modeling the free motion of a rigid body with a cavity filled by a viscous liquid. Zhukovskiy's Theorem states that in the limit  $t \rightarrow \infty$ , the relative fluid velocity tends to zero and the rigid velocity of the full structure tends to a steady rotation around one of the principle axes of inertia. We

---

show that every (reasonable) weak solution is subject to Zhukovskiy's Theorem – i.e. that it applies to finite-energy initial data of arbitrary size. In particular, the weak solutions constructed in [2] are of this type.

For this system, the kinetic energy provides a strict Lyapunov functional for (regular) solutions, whereas total angular momentum is preserved. One of the main points in the proof is to show that despite this conservative aspect and in the absence of stability, weak solutions become regular eventually. For large-time trajectories, a suitable version of LaSalle's invariance principle then applies.

## References

- [1] Karoline Disser. Asymptotic behaviour of a rigid body with a cavity filled by a viscous liquid. WIAS Preprint 1958, 2014.
- [2] Ana L. Silvestre and Takeo Takahashi. On the motion of a rigid body with a cavity filled with a viscous liquid. Proc. Roy. Soc. Edinburgh Sect. A, 142(2):391423, 2012.

# Regularity of weak solutions to the full Navier-Stokes-Fourier system

Eduard Feireisl

Czech Academy of Science, Prague, Czech Republic  
feireisl@math.cas.cz

We discuss the problem of regularity and conditional regularity of weak solutions to the full Navier-Stokes-Fourier system. Applications will be given to convergence problems of the associated numerical schemes.

---

# Stability of time-dependent Navier-Stokes flow and algebraic energy decay

Toshiaki Hishida

Graduate School of Mathematics, Nagoya University  
Nagoya 464-8602 Japan  
hishida@math.nagoya-u.ac.jp

Let  $V = V(x, t)$  be a given time-dependent Navier-Stokes flow of an incompressible viscous fluid in the whole space  $\mathbb{R}^n$  ( $n = 3, 4$ ). As important examples of this basic flow  $V$ , we have the following in mind: forward self-similar solution, time-periodic solution and global mild solution of the Cauchy problem. It is thus reasonable to assume that  $V \in L^\infty(0, \infty; L^{n, \infty}) \cap C_w([0, \infty); L^{n, \infty})$ , where  $L^{n, \infty}$  denotes the weak- $L^n$  space. The energy stability of small  $V$  in this class with respect to any initial disturbance in  $L^2_\sigma$  has been investigated by [1]. In this presentation we would like to discuss structure of energy decay rate of disturbance for  $t \rightarrow \infty$ .

This talk is based on a joint work with Maria E. Schonbek (University of California, Santa Cruz).

## References

[1] G. Karch, D. Pilarczyk and M.E. Schonbek,  $L^2$ -asymptotic stability of mild solutions to the Navier-Stokes system of equations in  $\mathbb{R}^3$ , Preprint, arXiv:1308.6667.

# Serfati's approach to vortex patches

James P. Kelliher

University of California Riverside  
kelliher@math.ucr.edu

We discuss Ph. Serfati's proof of the persistence of regularity of the boundary of a two-dimensional vortex patch. This proof, published in a four-page paper in 1994, was preceded by two earlier proofs of the same result, one by Chemin and the other by Bertozzi and Constantin. Though Serfati's proof clearly owes much to each of these prior proofs, it is elegant, elementary, and quite novel. Some of its ideas, buried for two decades, have applications to problems of current interest,

---

in particular to the propagation of local regularity. We speak of this and other applications. This is a report on joint work with Hantaek Bae of UC Davis.

## **The Kadomtsev-Petviashvili II equation in 3d**

Herbert Koch

Bonn University,  
Mathematisches Institut,  
koch@math.uni-bonn.de

The Kadomtsev-Petviashvili II equation describes wave propagating in one direction with weak transverse effect. I will explain the proof of global existence and scattering for three space dimensions. The key estimates are bilinear  $L^2$  estimates and a delicate choice of norms. This is joint work with Junfeng Li.

## **Weak time-periodic solutions to the Navier-Stokes equations in the three-dimensional whole-space with a non-zero drift term: Asymptotic profile at spatial infinity**

Mads Kyed

Institut für Mathematik  
Heinrich Plett Str. 40  
D-34132 Kassel, Germany  
mkyed@mathematik.uni-kassel.de

Results on the asymptotic structure at spatial infinity of solutions to the time-periodic Navier-Stokes system in the three-dimensional whole-space with a non-zero drift term will be presented. We will introduce a large class of weak time-periodic solutions and establish an asymptotic expansion as  $|x| \rightarrow \infty$  for the members of this class. The asymptotic profile will be explicitly identified in terms of the Oseen fundamental solution. Moreover, a pointwise estimate will be given for the remainder term. This estimate will manifest a parabolic wake region in the solution.

---

# The limit of small viscosity and small elastic response for the second-grade fluid equations

Milton da Costa Lopes Filho

Universidade Federal do Rio de Janeiro, Brasil  
clavyus@gmail.com

In this talk we consider the second-grade fluid system with viscosity  $\nu$  and elastic response parameter  $\alpha$ , in a two-dimensional smooth bounded domain with smooth initial data and no-slip boundary conditions. For a suitably defined converging family of initial data, we consider a corresponding family of solutions to these equations depending on these parameters. We examine the limits of vanishing  $\alpha$  and  $\nu$ , obtaining, for certain regimes, convergence to solutions of the incompressible Euler equations, while in other regimes, we obtain sharp conditions for convergence analogous to Kato's criterion. This talk is based on joint work with H. Nussenzweig Lopes, E. Titi and A. Zang.

## Existence, uniqueness and approximation of some diffusive viscoelastic flows

Mária Lukáčová-Medvid'ová

Johannes-Gutenberg University, Mainz  
Staudingerweg 9, 55 129 Mainz,  
lukacova@uni-mainz.de

In this talk we will present our recent results on the so-called diffusive Peterlin model, which can be obtained by a suitable approximation of a nonlinear bed-spring force. Consequently we obtain a macroscopic model for the conservation of mass, momentum and time evolution of the conformation stress tensor, which takes into account also diffusive effects.

We will study existence and regularity of weak solutions in two and three space dimensions and show corresponding uniqueness for more regular solutions, cf. [1]. Having obtained uniqueness and regularity result we propose a numerical scheme for the approximation of the diffusive Peterlin model that is based on the characteristics finite element method, cf. [3,4]. Finally we will show that using  $P_1/P_1stab/P_1$  finite element approximation for the velocity, pressure and conformation tensor we obtain the first order error estimates, cf. [2].

---

The present work has been supported by the German Science Foundation (DFG) under IRTG 1529 “Mathematical Fluid Dynamics” and realized in collaboration with H. Mizerová, Š. Nečasová, M. Renardy, M. Tabata, H. Notsu and B. She.

## References

- [1] M. Lukáčová-Medvid’ová, H. Mizerová, Š. Nečasová: *Global existence and uniqueness results for the diffusive Peterlin viscoelastic model*, Preprint, University of Mainz, 2014.
- [2] M. Lukáčová-Medvid’ová, H. Mizerová, H. Notsu, M. Tabata: *Error estimates of a pressure-stabilized characteristics finite element scheme for the diffusive Oseen-Peterlin model*, in preparation.
- [3] H. Notsu, M. Tabata: *A single-step characteristic-curve finite element scheme of second order in time for the incompressible Navier-Stokes equations*, J. Sci. Comput. 38 (2009), no. 1, 1-14.
- [4] H. Notsu, M. Tabata: *Error estimates of a pressure-stabilized characteristics finite element scheme for the Oseen equations*, WIAS Discussion Paper 2013, no. 1, 1-19.

# A sort of continuous dependance on data for a suitable weak solution to the Navier-Stokes Cauchy problem

Paolo Maremonti

Second University of Naples  
via Vivaldi, 43 Caserta  
paolo.maremonti@unina2.it

As it is known, in [1] is proved the partial regularity for suitable solutions to the Navier-Stokes Cauchy problem. In particular, it is proved that a suitable weak solution becomes “regular” in the exterior of a suitable ball  $B_R$ . The aim of our study is to investigate, for a suitable weak solution, a sort of continuous dependence. More precisely, we are interested to point out a bound and the behavior in  $t$ , in a neighborhood of  $t = 0$ , of  $L^\infty(\mathbb{R}^3 - B_R)$ -norm of solutions and also the pointwise behavior of the solutions for large  $|x| > R$ , provided that a one exists for the initial data.

The results are part of the papers [2,3].

---

## References

- [1] Caffarelli L., Kohn R. and Nirenberg L., Partial regularity of suitable weak solutions of the Navier-Stokes equations, CPAM, 35 (1982).
- [2] Crispo F. and Maremonti P., A remark on the partial regularity of a suitable weak solution to the Navier-Stokes Cauchy problem, forthcoming.
- [3] Crispo F. and Maremonti P., On the spatial-time decay of a suitable weak solutions to the Navier-Stokes Cauchy problem, forthcoming.

## Mathematical cocktails: weak solutions

Piotr Mucha

University of Warsaw, Poland  
p.mucha@mimuw.edu.pl

The aim of the talk is analysis of a model describing motion of chemically reacting heat-conducting gaseous mixtures, based on a modification of the compressible Navier-Stokes equations. The key point is thermodynamical well posedness of the studied model. I would like to focus on presenting the detailed approximation scheme for the full system. At least on the first steps.

The talk will base on joint results with Milan Pokorný and Ewelina Zatorska.

## Low Mach number limit and diffusion limit in a model of radiative flow

Šárka Nečasová

Institute of Mathematics of the Academy of Sciences of the Czech Republic,  
Žitná 25, 115 67 Praha, Czech Republic  
matus@math.cas.cz

We consider an asymptotic regime for a simplified model of compressible Navier-Stokes-Fourier system coupled to the radiation, when hydrodynamical flow is driven to incompressibility through the low Mach number limit. We prove a global-in-time existence for the primitive problem in the framework of weak solutions and for the incompressible target system and we study the convergence of the primitive system toward its incompressible limit. Moreover, we investigate the cases when

---

the radiative intensity is driven either to equilibrium or to non-equilibrium diffusion limit, depending the scaling performed, and we study the convergence of the system toward the aforementioned limits. Finally, we consider a "semi-relativistic" model of radiative viscous compressible Navier-Stokes-Fourier system coupled to the radiative transfer equation extending the classical model introduced in [3] and we study some of its singular limits (low Mach and diffusion).

## References

- [1] B. Ducomet, Š. Nečasová, Low Mach number limit in a model of radiative flow, *J. Evol. Equ.* 14 (2014), **2**, 357–385.
- [2] B. Ducomet, Š. Nečasová, Diffusion limits in a model of radiative flow, *Ann Univ Ferrara* DOI 10.1007/s11565-014-0214-3.
- [3] B. Ducomet, E. Feireisl, Š. Nečasová, On a model of radiation hydrodynamics. *Ann. I. H. Poincaré-AN* **28** (2011), 797–812.
- [4] B. Ducomet, Š. Nečasová, Singular limits in a model of radiative flow, Preprint 2014.

# Maxwell-Stefan Diffusion in Reactive Multicomponent Flows

Jan Prüss

Martin-Luther-Universität Halle-Wittenberg,  
D-06120 Halle, Germany  
jan.pruess@mathematik.uni-halle.de

We consider one-phase reactive multi-component flows where diffusion is modeled by the Maxwell-Stefan approach. Based on abstract results on quasi-linear parabolic evolution equations, we show local well-posedness of the system in an  $L_p$ -setting, and construct the resulting local semiflow. The equilibria of the system are identified and are shown to be exponentially, employing the generalized principle of linearized stability. Further, the free energy of the system defines a strict Ljapunov-functional, and by this we show that solutions which do not develop singularities converge to a single equilibrium. (Joint work with M. Wilke)

## References

- [1] V. Giovangigli: *Multicomponent flow modeling*. Birkhäuser, Basel (1999).

---

[2] D. Bothe, W. Dreyer: *Continuum thermodynamics of chemically reacting fluid mixtures*, preprint 2014.

[3] M. Herberg, M. Meyries, J. Prüss, M. Wilke: *Reaction-diffusion systems of Maxwell-Stefan type with reversible mass-action kinetics*, preprint, submitted 2013.

[4] J. Prüss, G. Simonett, and R. Zacher: *On convergence of solutions to equilibria for quasilinear parabolic problems*, *Journal of Differential Equations*, 246, 3902–3931 (2009).

## On a global well-posedness of strong dynamics of Incompressible Nematic Liquid Crystals in $R^N$

Yoshihiro Shibata

Waseda University, Dept. Maths and RISE  
Tokyo 169-8555, Japan  
yshibata@waseda.jp

I will talk about some technique how to prove a global well-posedness of strong solutions at least small initial data for some system of parabolic equations by some combination of maximal  $L_p$ - $L_q$  regularity with  $L_p$ - $L_q$  decay estimate in a unbounded domain. This is a joint work with Maria Schoenbek.

## Remarks on the Strichartz estimates for the rotating incompressible fluids

Ryo Takada

Mathematical Institute, Tohoku University  
Sendai 980-8578 Japan  
ryo@m.tohoku.ac.jp

In this talk, we consider the Strichartz estimates for the linear propagator  $e^{\pm it \frac{D_3}{|D|}}$  generated by the Coriolis force  $e_3 \times u$ :

$$\left\| e^{\pm it \frac{D_3}{|D|}} P_0 f \right\|_{L_t^q L_x^r} \lesssim \|f\|_{L_x^2}.$$

Here  $P_0 f$  is a frequency projection operator defined by  $\widehat{P_0 f} = \varphi(|\xi|) \widehat{f}$ ,  $\varphi \in C_c^\infty((\frac{1}{2}, 2))$ . In our previous work [1], we derived the optimal admissible range

---

$1/q + 1/r \leq 1/2$  for the above space-time estimates. In this talk, we shall give an improvement for the admissible relation under the action of differential operators. Also, we shall discuss a generalization to the rotating stratified fluids.

## References

[1] Y. Koh, S. Lee and R. Takada, *Strichartz estimates for the Euler equations in the rotational framework*, J. Differential Equations **256** (2014), 707–744.

# Free and controlled particles in viscous incompressible flows

Marius Tucsnak

University of Lorraine,  
Institut Elie Cartan, BP 70239, 54506 Vandoeuvre-lès-Nancy Cedex  
`marius.tucsnak@lorraine.fr`

We study the nonlinear system coupling PDEs and ODEs which models the motion of rigid bodies in a viscous incompressible fluid. We first consider the case in which particles are *free*, i.e., under the influence of hydrodynamic forces only, and we recall some of the methods used to solve the difficulties encountered in the mathematical analysis of this problem (the main one being the presence of a free boundary).

We next study two control problems for solids moving in a viscous incompressible fluid. In the first one, the aim consists in steering the bodies to prescribed positions by means of exterior forces acting on them. We show, in particular, that in the presence of control forces we can obtain existence and uniqueness results which are better than in the uncontrolled case. The second control mechanism consists in appropriate deformation of the solids: this is the swimming problem.

---

# On the non-homogeneous Navier-Stokes equations

Werner Varnhorn

Institute of Mathematics, Kassel University  
Heinrich-Plett-Str. 40, 34109 Kassel  
varnhorn@mathematik.uni-kassel.de

The investigation presented here is a joint work with Reinhard Farwig (TU Darmstadt) and Hermann Sohr (U Paderborn): Consider a bounded domain  $\Omega \subseteq \mathbb{R}^3$  with smooth boundary  $\partial\Omega$ , a time interval  $[0, T)$ ,  $0 < T \leq \infty$ , and in  $[0, T) \times \Omega$  the non-homogeneous Navier-Stokes system

$$u_t - \Delta u + u \cdot \nabla u + \nabla p = f, \quad u|_{t=0} = u_0, \quad \operatorname{div} u = k, \quad u|_{\partial\Omega} = g,$$

with sufficiently smooth data  $f, u_0, k, g$ . In this general case there are mainly known two classes of weak solutions, the class of global weak solutions, similar as in the well known case  $k = 0, g = 0$ , which need not be unique, and the class of local very weak solutions, see [1], [2], [3], which are uniquely determined, but need neither have differentiability properties nor satisfy the energy inequality. Our aim is to introduce a new class of local strong solutions for the general case  $k \neq 0, g \neq 0$ , satisfying similar regularity and uniqueness properties as in the known case  $k = 0, g = 0$ . For slightly restricted data this class coincides with the corresponding class of very weak solutions yielding new regularity results. Further, through the given data we obtain a control on the interval of existence of the strong solution (compare [4], [5]).

## References

- [1] H. Amann, Nonhomogeneous Navier-Stokes equations with integrable low-regularity data, *Int. Math. Ser.*, Kluwer Academic/Plenum Publishing, New York, 2002, 1–26.
- [2] R. Farwig, G. P. Galdi, H. Sohr, A new class of weak solutions of the Navier-Stokes equations with nonhomogeneous data, *J. Math. Fluid Mech.* 8 (2006), 423–444.
- [3] R. Farwig, H. Kozono, H. Sohr, Very weak, weak and strong solutions to the in-stationary Navier-Stokes system, *J. Nečas Center for Mathematical Modeling, Lecture Notes*, Vol. 2, P. Kaplický, Š. Nečasová (eds.), pp. 15–68, Prague 2007.
- [4] R. Farwig, H. Sohr, W. Varnhorn, On optimal initial value conditions for local strong solutions of the Navier-Stokes equations, *Ann. Univ. Ferrara* 55 (2009), 89–110.

---

[5] R. Farwig, H. Sohr, W. Varnhorn, Extensions of Serrin's uniqueness and regularity conditions for the Navier-Stokes equations, *J. Math. Fluid Mech.* 14 (2012), 529–540.

## Low Mach Number Limit for the Multicomponent Reactive Mixture Model

Ewelina Zatorska

IAMM, University of Warsaw,  
ul Banacha 2, 02-097 Warszawa, Poland,  
e.zatorska@mimuw.edu.pl

This talk will be devoted to existence result for the low Mach number limit system obtained from the full compressible Navier-Stokes model for multicomponent reactive mixture. We will first present the compressible model [2] and explain the issue of global in time existence of solutions under special form of the stress tensor [3]. Then we will present derivation of the incompressible system with variable density, called Kazhikhov-Smagulov type model, for a heat-conducting binary mixture. Under special compatibility condition between the viscous tensor and the diffusive term we will prove the existence of global in time weak solutions [1]. The proof relies on the use of a special relative velocity, which is divergence free and which allows to reduce the coupling between particular subsystems. We will also mention possible generalizations of various constraints appearing in the system.

### References

- [1] D. Bresch, V. Giovangigli, E. Zatorska Full low Mach number limit for a model of gaseous mixture – existence of solutions. *in preparation*
- [2] V. Giovangigli. *Multicomponent flow modeling*. Modeling and Simulation in Science, Engineering and Technology. Birkhäuser Boston Inc., Boston, MA, 1999.
- [3] E. Zatorska. On a steady flow of multicomponent, compressible, chemically reacting gas. *Nonlinearity*, 24:3267–3278, 2011.

---

# Computational Theory of Function Spaces

Martin Ziegler

Technische Universität Darmstadt,  
64289 Darmstadt, Germany  
ziegler@mathematik.tu-darmstadt.de

---

## 4 Participants

---

**Abe Ken**, Nagoya University

**Abels Helmut**, Universität Regensburg

**Bolkart Martin**, TU Darmstadt

**Bothe Dieter**, TU Darmstadt

**Brandolese Lorenzo**, Université Lyon

**Caggio Matteo**, Academy of Sciences of the Czech Republic

**Cismas Emanuel**, Leibniz Universität Hannover

**Constantin Peter**, Princeton University

**Dalinger Alexander**, TU Darmstadt

**Danchin Raphael**, Université Paris

**Disser Karoline**, WIAS Berlin

**Egert Moritz**, TU Darmstadt

**Egger Herbert**, TU Darmstadt

**Farwig Reinhard**, TU Darmstadt

**Feireisl Eduard**, Academy of Sciences of the Czech Republic

**Förster Clemens**, Universität Leipzig

**Giga Yoshikazu**, Tokyo University

**Giga Mi-ho**, Tokyo University

**Guillod Julien**, University of Geneva

**Hieber Matthias**, TU Darmstadt

**Hishida Toshiako**, Nagoya University

**Iwabuchi Tsukasa**, Chuoh University

**Kato Tomoya**, Nagoya University

**Kalousek Martin**, Academy of Sciences of the Czech Republic

**Kelliher Jim**, University of California

**Koba Hajime**, Waseda University

---

**Koch Herbert**, Universität Bonn  
**Kozono Hideo**, Waseda University  
**Kyed Mads**, TU Darmstadt  
**Lopes Filho Milton**, Universidade Federal do Rio de Janeiro  
**Lukáčová Mária**, Johannes-Gutenberg Universität Mainz  
**Maekawa Yasunori**, Tohoku University  
**Maremonti Paolo**, Seconda Università degli Studi di Napoli  
**Meyer Stefan**, Martin-Luther-Universität Halle-Wittenberg  
**Miura Hideyuki**, Tokyo Institute of Technology  
**Mizerova Hana**, Johannes-Gutenberg Universität Mainz  
**Mori Naofumi**, Kyushu University  
**Mucha Piotr**, University of Warsaw  
**Nečasová Šárka**, Academy of Sciences of the Czech Republic  
**Nobili Camilla**, Universität Leipzig  
**Noboriguchi Dai**, Waseda University  
**Okabe Takahiro**, Hirosaki University  
**Prüss Jan**, Martin-Luther-Universität Halle-Wittenberg  
**Saal Martin**, TU Darmstadt  
**Sauer Jonas**, TU Darmstadt  
**Schade Katharina**, TU Darmstadt  
**Schmäche Christopher**, Universität Leipzig  
**Schmid Verena**, TU Darmstadt  
**Schumann Rainer**, Universität Leipzig  
**Seitz Tobias**, TU Darmstadt  
**She Bangwei**, Johannes-Gutenberg Universität Mainz  
**Shibata Yoshihiro**, Waseda University  
**Székelyhidi László**, Universität Leipzig  
**Suzuki Tomoyuki**, Tokyo University  
**Takada Ryo**, Tohoku University

---

**Takahashi Go**, Waseda University  
**Tarfulea Andrei**, Princeton University  
**Terasawa Yutaka**, Nagoya University  
**Tolksdorf Patrick**, TU Darmstadt  
**Tucsnak Marius**, Université de Lorraine  
**Ushikoshi Erika**, Tamagawa University  
**Varnhorn Werner**, Universität Kassel  
**Yanqiu Guo**, Weizmann Institute  
**Zatorska Ewelina**, University of Warsaw  
**Ziegler Martin**, TU Darmstadt